Whole Number Multiplication Algorithm

Question:

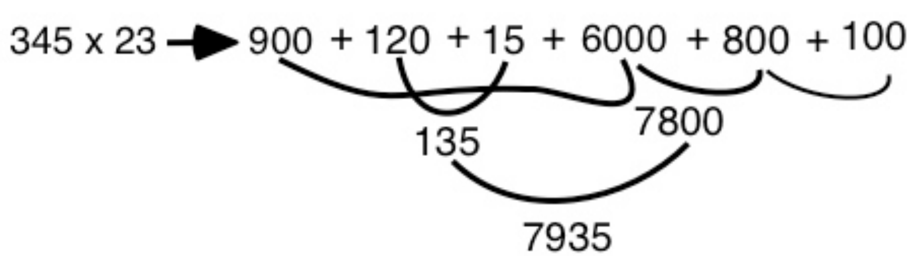
How can you get an answer to 23 x 345?

Possible answers to the question are:

1. Pull out the calculator and punch in the numbers.
2. Add 345 twenty-three times.
3. I'll show you what I was taught years ago. I didn't (and still don't) understand why the method works but it got the right answer according to the teacher.

```
  1
  1 1
  345
 x 23
------
 1035
 690
------
 7935
```

4. I saw a weird method in my kid's grade 5 class for doing big multiplications. It had a lot of numbers written down but I was making some sense of why the numbers were there. For example, I think 900 came from 3 x 300 and 135 came from 120 add 15. The method looked like this.



Response 1 indicates a lack of empowerment about doing arithmetic. It signals that the person is unable to do the arithmetic when a calculator (or similar device) is not available, thus leaving the person at the mercy of those who can do such things.

Response 2 indicates an ability to do multiplications based on a simplistic approach - repeated addition. That approach is reasonable for small numbers but it becomes cumbersome and tedious, and error prone for larger numbers.

Response 3 is what most adults were taught in elementary school. The method is efficient but does not reveal any understanding about why it works. It was taught because in the "good old days" part of the purpose of elementary school mathematics curricula was to train the mind to be an efficient and error free calculator (electronic calculating devices were not around/very expensive in those days).

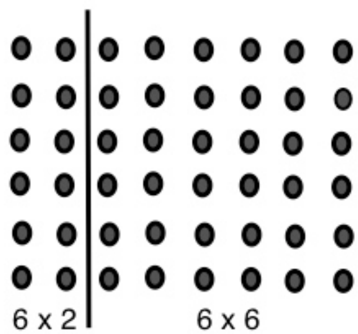
Response 4 provides insight into what is going on with the method. It is a slower procedure that that of response 3. However, it reveals conceptual matters that can lead to an understanding of why the method works. The “secret” to doing multiplication with big numbers is visually present but guidance would likely be needed to “see” it.

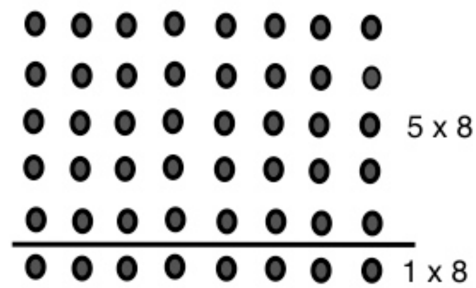
The Distributive Principle

The “secret” hinted at above that underlies efficient methods for multiplying with larger numbers is known as the distributive principle. It is as “old as the hills”. For example, the method Egyptians used (4000 years ago) to multiply involved the distributive principle.

If understanding a multiplication method for larger numbers is the teaching goal, then the distributive principle MUST BE understood.

What is the distributive principle? Consider the two diagrams below. Each represents an array model for 6 x 8. The diagrams and subsequent arithmetic illustrate the distributive principle in action.





The left diagram indicates the 6 by 8 array was cut into two smaller arrays: a 6 x 2 and a 6 x 6 by cutting the 8 columns into 2 and 6. The answer to 6 x 8 can be figured out by doing 6 x 2 and 6 x 6, getting 12 and 36, and then adding the multiplication results to obtain 48.

The rightmost diagram indicates the 6 by 8 array was cut into two smaller arrays: a 5 x 8 and a 1 x 8 by cutting the 6 rows into 5 and 1. The answer to 6 x 8 can be figured out by doing 5 x 8 and 1 x 8, getting 40 and 18, and then adding the multiplication results to obtain 48.

The distributive principle, in simple terms, says that you can cut a multiplication up into smaller multiplications, and add the results to obtain the answer to the multiplication.

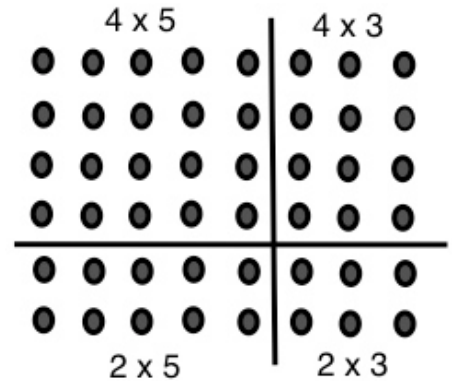
The diagrams and arithmetic can be represented symbolically using brackets.

$$6 \times 8 = 6 \times (2 + 6) = 6 \times 2 + 6 \times 6 = 12 + 36 = 48$$

$$6 \times 8 = (5 + 1) \times 8 = 5 \times 8 + 1 \times 8 = 40 + 8 = 48$$

Both rows and columns and columns can be cut. Here is an example.

The row was cut into 4 and 2. The column was cut into 5 and 3.



The diagrams and arithmetic can be represented symbolically using brackets.

$$6 \times 8 = (4+2) \times (5+3) = 4 \times 5 + 4 \times 3 + 2 \times 5 + 2 \times 3 = 20 + 12 + 10 + 6 = 48$$

The Multiplication Algorithm (transparent distributive principle)

Consider 8 x 56. There are many ways to cut the numbers in order to obtain an answer.

For example, you could think: 8 x 10 + 8 x 20 + 8 x 20 + 8 x 6.

For example, you could think: 5 x 40 + 3 x 40 + 5 x 16 + 3 x 16

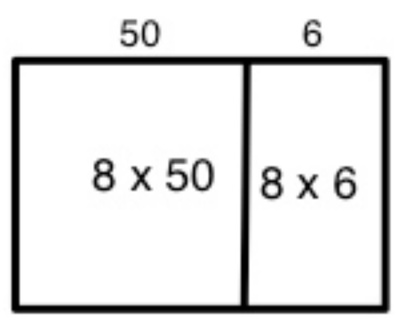
Normally, the most efficient way to cut is along place value positions and maximizing the place value cuts. For 8 x 56, this means thinking: 8 x 50 + 8 x 6. The work can be written horizontally or vertically.

Horizontally: 8 x 56 = 8 x 50 + 8 x 6 = 400 + 48 = 448

Vertically:

```
 56
 x 8
----
400  <---  8 x 50
 48  <---  8 x 6
----
448
```

A rectangle diagram can be used to indicate the situation. It is a short way of “showing” the array without drawing the dots. Here is the rectangle for the above horizontal and vertical work.

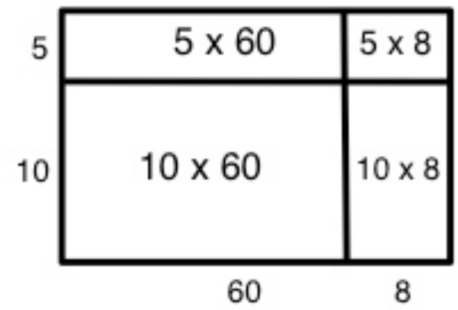


One more example.

Consider 15 x 68. The cuts that maximize place value positions are:

- Cut 15 into 10 and 5
- Cut 68 into 60 and 8

The rectangle diagram looks as shown here. The answer would be obtained by adding 600, 300, 80, and 40 in any order.



Refer to: Grade 4 Multiplication algorithm if more help needed.